Published in: Selected articles about the world of music arts and education. Volume II. UNESCO Chair in Musical Arts and Education in Life-Long Learning at the Moscow State University of Education, pp. 150-169. – Moscow 2016: Moscow State University of Education (ISBN 978–5–4263–0352–2), 310 p.

METHOD OF VISUAL PRESENTATION OF ALL SHORTEST MODULATION PATHS FROM ANY KEY TO ANY KEY: LEARNING OF MODULATION SYSTEMS IN THE CONTEXT OF PROFESSIONAL MUSIC-THEORETICAL EDUCATION

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Abstract. *The article considers important for pedagogy of music education the solution to the problem of finding all possible shortest modulation paths from any key to any key in conformity with all possible modulation systems. It presupposes a modulation via a chord serving as a common one for neighboring keys in the process of modulation and/or via a sequence of two chords with the same tonal center, but of the different tonal types (major vs. minor). A concept of mutual relations between both close and distant keys is set out in the article in terms of the psychology of aesthetic perception. A method to solve the problem of finding modulation paths is also provided. The method is based on visual presentation using a "graphic induction".*

Keywords: *harmony, modulation, modulation plans, visual presentations, pedagogy of music education.*

This article was written in 1974 in Russian, and for reasons beyond my control, it was only published in Russian 40 years later. The article therefore only has references to the sources from the Russian music theory literature. After extensive research in analogues of the English- and German languages, I have been unable to find anything similar which solves the problem to the same extent.

The article concerns the problems of both theoretical musicology and music education pedagogy. The method of mastering a most chronophageous section of the harmony course can be applied in the system of professional music education.

The logic of relations between keys in connection with the building of all possible shortest modulation paths was scrutinized both in scientific research and in the sources on methodology of teaching music-theoretical disciplines. Russian musicology views this problem as the problem of "kinship" (or "relationship") between the various keys. Though I do not agree with this term, I will use it for a while, due to its traditional use in Russian music-theoretical literature and in Russian harmony textbooks. A detailed analysis of this problem can be found in the monograph by L. Mazel [1, pp. 344–410]. A faultless mathematical solution as applied to the modulation system by Rimsky-Korsakov was proposed by M. Iglitsky [2, pp. 190–205].

The problem in question is of interest not only from the academic point of view. Since the probability of a certain modulation in a certain harmonic style has a particular meaning, the meaning of modulation remains vague unless this probability is evaluated. For the sake of comprehending meanings borne by the means of music expressiveness, music-theoretical disciplines are studied. I propose the following:

a) the conception of relations between various keys from the point of view of aesthetic perception psychology;

b) the method of mastering these relations in music education on the basis of graphic presentation of ***all possible modulation*** systems which cater for modulation via a common chord and/or via a sequence of two chords with the same tonal center, but of the different tonal types ("eponymous juxtaposition"), and of disclosing in them ***all possible shortest modulation paths***.

The logical apparatus I am going to use can be easily understood not only by university students, but also by students from secondary professional education institutions. It is evident from my experience of utilizing it at the music college of Tiraspol (Moldova), and in Moscow at a Secondary specialized music school for highly gifted children named after Gnessins (for students of 10 years of age).

First and foremost, let me note that the issue of degree of relationship between various tonal centers is often formulated erroneously for the fact that the eponymous juxtaposition as well as modulation itself can perform different functions has been ignored in a special literature. I will call them developing and restraining for convention's sake.

Transition from major to eponymous minor followed by active modulation movement is an example of a developing juxtaposition. Transition from minor to eponymous major usually followed by a caesura of a certain depth is an example of a restraining juxtaposition. The latter can be explained by a special acoustic role of major (hence "Picardy thirds" in minor cadences of baroque composers).

Modulations that form a chain of keys leading to a culmination refer to the developing modulations. In classical music they are typically inclined towards a subdominant key. The restraining modulation, on the contrary, leads to a caesura and is dominant-oriented.

The reasons for this are the following. The idea of a certain modulation from the point of view of harmony (irrespective of form) consists of interrupting "harmonic inertia" at the level of text, but of preservation "harmonic inertia" at the level of language. I have published a special investigation devoted to this theme [3, pp. 212–231]. Herewith I will only note that perception inertia interruption at the level of text is inertia interruption at the level of inner relationship between language signs, while perception inertia interruption at the level of language occurs at the level of external relationship between language signs. Here the external relationship assumes the relationship between signs from different texts, while their functions in these texts are the same.

In particular, I would like to put forward the supposition that, in relation to the aesthetic perception, the interruption of inertia of perception at the level of text should be accompanied by inertia manifestation at the level of language, and vice versa.

As far as harmonic inertia interruption at the level of text is concerned, it is confirmed by any kind of modulation. Inertia retention at the level of language means that, of two possible trends (subdominant-oriented or dominant-oriented), the one involving a lesser inertia interruption is opted for. Let me explain this through the so-called complete harmonic cadence and distribution of keys in classical compositions. I will return to comparing different types of modulations.

Subdominant (further S) is used in Russian tradition with reference to a family of chords which include the 6th scale degree, and keys with these chords as tonal centers. Eg. when in C major, this family consists of D minor/D flat major, F major/F minor, and A minor/A flat major. Dominant (further D) refers to a family of chords which include the 7th scale degree, and keys with these chords as tonal centers. Eg. when in C major, this family consists of E minor, and G major. When in C minor, the subdominant family of keys consists of D flat major, F minor, and A flat major. The dominant family of C minor consists of E flat major, G minor/G major, and B flat major.

A complete harmonic cadence is T-S-D-T (tonic-subdominant-dominant-tonic), while a distribution of keys in classical compositions is T-D-S-T. The difference is that in the harmonic cadence, first and foremost, melodic relations can be heard (due to tight temporal proximity of harmonic means), whereas in the distribution of keys acoustic relations are primary (due to remoteness of tonal centers of different keys within a composition).

In a cadence, transition from T to S interrupts the melodic inertia of the tonal center to a lesser extent than transition from T to D (the fifth degree of the subdominant is the same as a root of the tonic chord while in the dominant, a root of the tonic chord is replaced by the leading note). Besides which, at least in a major, the extent of tonic-to-subdominant melodic inclination (of 6th scale degree to 5th) is evidently lower than that of dominant-to-tonic melodic inclination (of 7th scale degree to 1st).

On the contrary, the extent of tonic-to-subdominant melodic inclination (availability of a leading note to the subdominant-root in the form of the 3rd degree of the scale) is higher than that of melodic inclination of a tonic chord to a dominant.

It is more complicated in minor. Its detailed analysis is a separate issue. Here I will concentrate on the methodology of explaining main trends to learners. More details on the topic can be found in a different work of mine [4, pp. 62–75]. With regard to the distribution of keys in a tonal composition, the transition from the main key to a dominant, realized far from the initial tonic within a composition, actually interrupts the acoustic inertia to a lesser extent than the transition to the subdominant key.

The reason for this phenomenon is that the roots of the tonic of both keys (T and D) are neighbors in a harmonic series as the 1st and the 2nd overtones (for example the rout of the tonic of C major as the main key and of G major as a dominant key in relation to C major). The 1st overtone belongs to the same pitch class as a basic pitch, that is to say that the dominant root is generated by the basic pitch. On the other hand, the root of F major tonic (as a subdominant key for C major) is situated at the 11th place within a harmonic series from C and has no acoustic relationship to C. In this way, the leading relations in restraining modulation are acoustic. In developing modulation the acoustic relations do not work. However, functionally mixed modulations are also possible. For example, the appearance of the second theme in Beethoven's "Sonata Pathétique" is concurrently a restraining modulation in a relative key and a developing juxtaposition (C minor – E flat minor). All these should be kept in mind when examining the systems of relationship between keys.

Thus, it can be said that the keys of first degree relationship in relation to the given one are the ones that interrupt the inertia of the given tonal center to the least extent from the point of view of a certain style (or even a certain work). The hallmark of minimum inertia interruption for developing modulations is the frequency of their utilization interiorly to the static parts of form (for example, in the first subject group of sonata-allegro form if any modulation takes place in it).

The hallmark of restraining modulations that marginally interrupt tonal inertia is the frequency of their utilization in transitions "via caesura" to secondary static form sections (for example in transition from the first subject group to the second subject group in sonata-allegro form).

In this sense, dynamic form sections are not exemplary. They are not only classical developments, but also contrastive juxtapositions "via caesura", like, for instance, between the first and the second movements of a sonata or a symphony. We can see that this definition is very conventional and it is to be specificated in every particular case.

As for other kinship relations, they are, on the contrary, to be determined clearly. The second degree is the first one as related to the first degree; the third degree is the first as related to the second degree; and so on.

The condition for kinship reciprocity put forward by L. Mazel is believed to be logically correct, but practically unnecessary. I mean that as long as a certain X is in the first degree of kinship to a certain Y, this Y is not necessarily in the first degree of kinship to X. The analogy with kinship relationship between people does not seem to be convincing.

Strictly speaking, L. Mazel is absolutely right when stating that the term "kinship" itself assumes reciprocity. I use this term here due to tradition. In fact, it would be more correct to say: first degree modulation, second degree modulation, etc. A modulation is not a static relation between keys, but a dynamic one. Movement in a music flow is unidirectional: from the past to the future.

When L. Mazel says that, in principle, it is possible to return to the original key via the same intermediate keys within which the modulation occurred, it is a progressing modulation. In the case of modulation "via caesura", it is invariably different. In the classical sonata-allegro form, there is no modulation from the major to the relative key or from the major to a subdominant key, in the course of transition from the first subject group to the second one, while backward movement is typical. Apparently, a different analogy is appropriate here: movement along a river flow or against a river flow (L. Mazel proposed it as well). The difference in the metabolic cost arising in this case is similar to the difference in sensations from the extent of tonal inertia interruption. The same concerns the progressing modulation, though to a lesser extent.

For better comprehension of the arising abundance of modulation systems, I propose the method of building up visual schemes of relationship between various keys with indication of all possible shortest modulation paths. Conventionally, I call it graphic induction.

It should be noted that in logic inductive reasoning is a reasoning that derives general principles from specific premises. In mathematic induction is a way of proving that verifies a given statement for all natural numbers on the basis of an argument that equals the first natural number; then a supposition is made that a function is valid if the argument is n; then it is proved that it is also valid if the argument is n+1. This is sufficient to consider the function valid at any argument value.

In the case in question I will put forward the method of visual presentation of the first unit in a modulation chain, as well as the method of visual presentation of any n-plus-one unit. Equal temperament makes this chain finite.

Let me demonstrate two models as examples: a symmetrical modulation system (satisfying the requirement of kinship reciprocity) and non-symmetrical one. The system by Rimsky-Korsakov [5, pp. 69–90] will represent the symmetrical system (though, of course, different systems may well substitute it). The non-symmetrical system will be represented by major-minor system of the following type: in the course of a modulation from a major, the keys eponymous to an initial key (as for instance C major and C minor) and to a dominant key (except for the ones utilized by Rimsky-Korsakov) will be considered of closely related, as well as a key of the 6th lower scale degree; in the course of a modulation from minor, the keys (except for the ones utilized by Rimsky-Korsakov) eponymous to major keys of diatonic kinship (a developing juxtaposition) will be considered, as well as the key eponymous to an initial one (a restraining juxtaposition). The diatonic kinship means the kinship between the main key and the keys, which tonic triads consist of the diatonic scale degrees of the main key.

The key eponymous to a subdominant is not to be considered closely related for the reason of controversies between the development (a subdominant) and the restraining (major).

Such a system more or less corresponds to the modulation in music of conventional-romantic stylistics. Not every impact of development and restraining is controversial.

If the restraining comes psychologically ahead of the development (development takes place where restraining is expected), such combination promotes modulation movements towards further keys (inertia interruption as manifestation of inertia of a higher level). Reverse combination tends to stop the modulation process (inertia interruption at the same level without recovery at another level, but all this, naturally, is limited by a certain conditional language), hence to be avoided.

A field of keys will be used as a graphic billet (scheme 1). Let us imagine all keys oriented along the Cartesian coordinate system in the way in which X-line (x) indicates the difference of one key signature between neighboring keys, and the Y-line (y) indicates the difference of three key signatures.

One might optionally select another function, for example $y=x\pm4$, in which there is even more logic: both on the X-line and on the Y-line, keys would have been in relations of fifths (plus 4 sharps = major dominant in minor; minus 4 flats = minor subdominant in major). However, I preferred $y=x\pm3$ as a more demonstrative one (on the Y-line, keys happen to be in eponymous relations: C major – C minor, etc.).

The tonal field shown in scheme 1 is a visual presentation of such a function, provided that it is envisaged that every key may be the center of a new coordinate system with the function $f(x)=x\pm3$.

Major scales are indicated by the capital letters, minor keys are indicated by the lowercase letters (eg. “Eb” means E flat major, “c#” means C sharp minor).

Scheme 1:

b	f♯	c♯	g♯	d♯	a♯	e♯	b♯	f𝄪
D	A	E	B	F♯	C♯	G♯	D♯	A♯
d	a	e	b	f♯	c♯	g♯	d♯	a♯
F	C	G	D	A	E	B	F♯	C♯
f	c	g	d	a	e	b	f♯	c♯
A♭	E♭	B♭	F	C	G	D	A	E
a♭	e♭	b♭	f	c	g	d	a	e
C♭	G♭	D♭	A♭	E♭	B♭	F	C	G
c♭	g♭	d♭	a♭	e♭	b♭	f	c	g
E𝄫	B𝄫	F♭	C♭	G♭	D♭	A♭	E♭	B♭
e𝄫	b𝄫	f♭	c♭	g♭	d♭	a♭	e♭	b♭
G𝄫	D𝄫	A𝄫	E𝄫	B𝄫	F♭	C♭	G♭	D♭

Also, I propose a numerical variant of the field of keys, where numbers indicate not the amount of key signatures in a certain key, as might appear at first sight, but the difference in key signatures between the box with naughts and the rest (in particular cases, when there is C major and A minor in the box with naughts, numbers also indicate the amount of key signatures in a certain box, major key being always below, and minor relative key being always above).

Scheme 2 shows the numerical variant of the field of keys (in concise form).

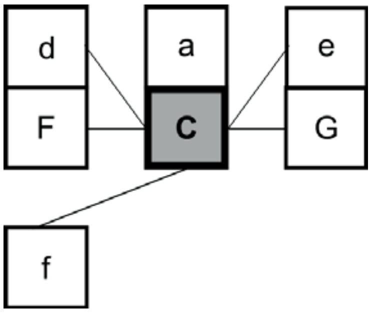
Scheme 2:

+3	+4	+5	+6	+7	+8	+9
+3	+4	+5	+6	+7	+8	+9
0	+1	+2	+3	+4	+5	+6
0	+1	+2	+3	+4	+5	+6
-3	-2	-1	0	+1	+2	+3
-3	-2	-1	0	+1	+2	+3
-6	-5	-4	-3	-2	-1	0
-6	-5	-4	-3	-2	-1	0
-9	-8	-7	-6	-5	-4	-3
-9	-8	-7	-6	-5	-4	-3

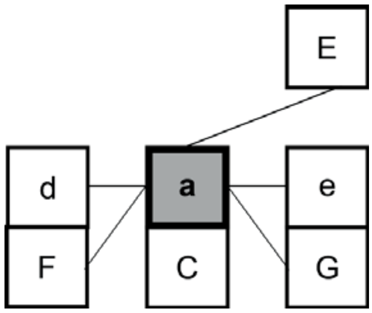
Let us turn to the practical utilization of the graphic induction. If we choose from the field of keys (see scheme 1) a certain key as the initial one in the modulation process and draw lines from it to intended keys of the first degree of relationship and then draw lines from each of these closely related keys to the keys of the same first degree of relationship but this time towards each of those of closely related, we will get the shortest paths leading to the second degree of relationship. To get the shortest paths to the third degree of relationship, we are to act in the same manner but with the keys of the second degree of relationship. How can it be performed in practice?

of demonstrativeness, start from C major (scheme 3). Then we cut out from the chart obtained rectangles with the names of keys and a stencil remains that symbolizes movement towards closely related keys. We do not need here a separate stencil for movement from the initial minor, for the system by Rimsky-Korsakov is symmetric and the minor stencil is the major one inverted upside down (scheme 4).

Scheme 3:



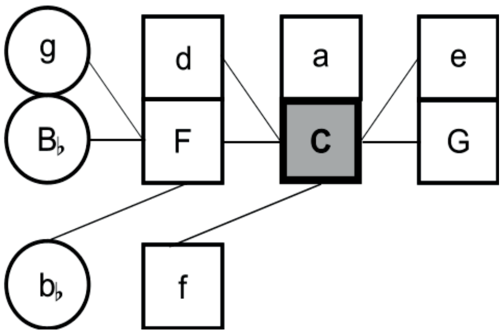
Scheme 4:



Applying these de facto two stencils on the parts of the field of keys that correspond to the keys F major (scheme 5), G major (scheme 6), D minor (scheme 7), A minor (scheme 8), E minor (scheme 9), and F minor (scheme 10) and transferring the result into a separate chart, we get a group of keys of the second degree with all shortest modulation paths that lead there from the initial key.

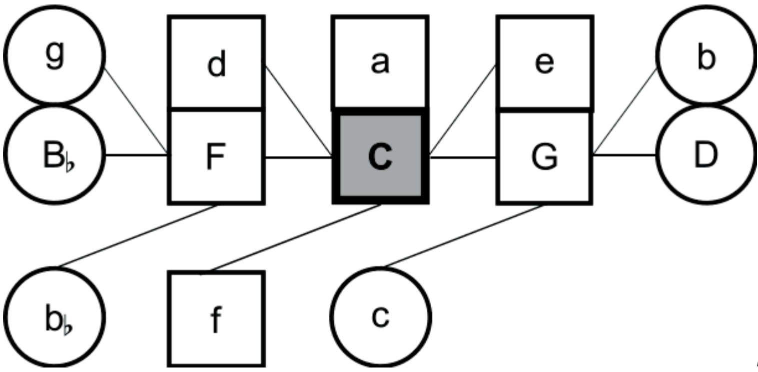
The cases of cut-out windows fitting closely related keys (first degree) are ignored (to avoid being at a stop or moving backward), and all new keys are symbolized by circles.

Scheme 5:

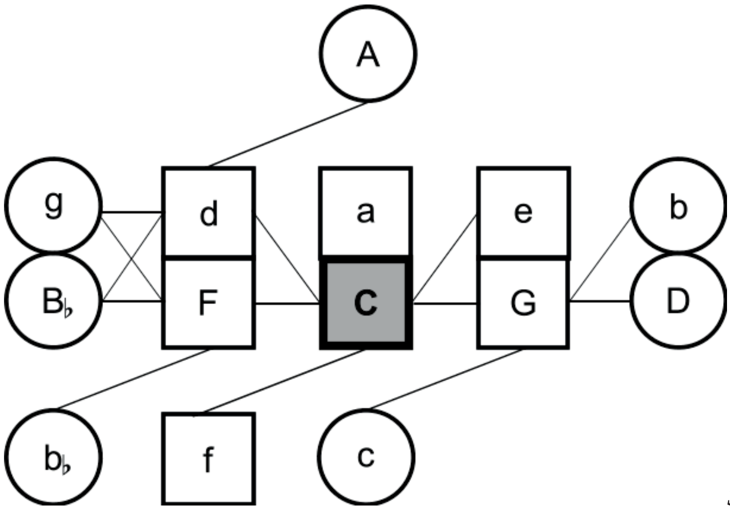


Let us take the system by Rimsky-Korsakov and, for the sake

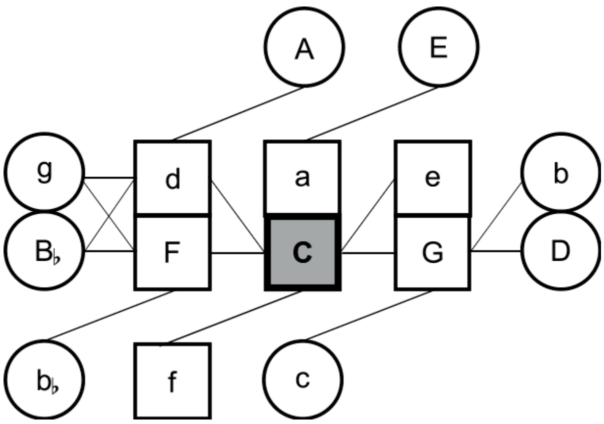
Scheme 6:

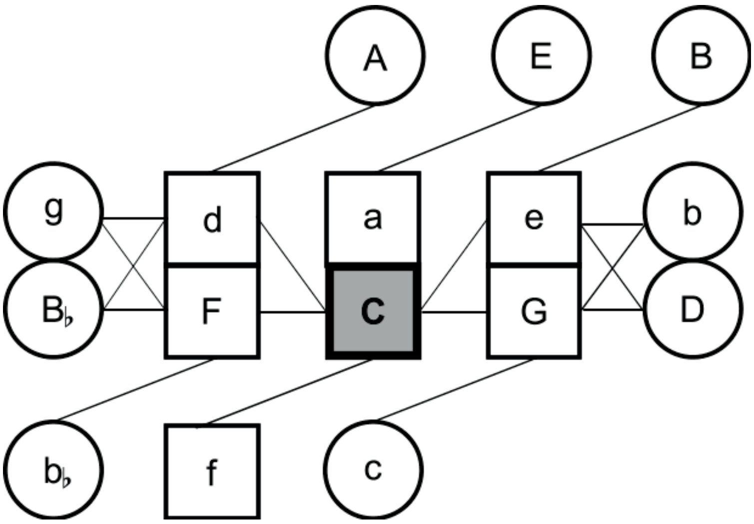


Scheme 7:

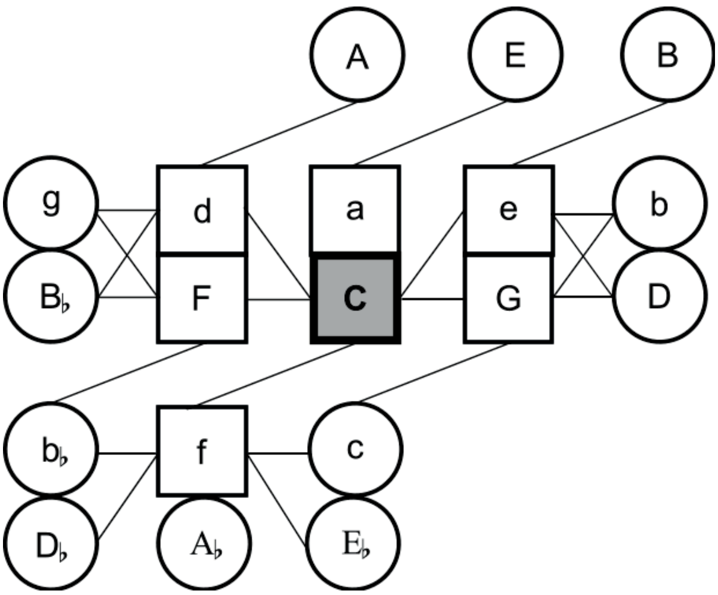


Scheme 8:





Scheme 10:



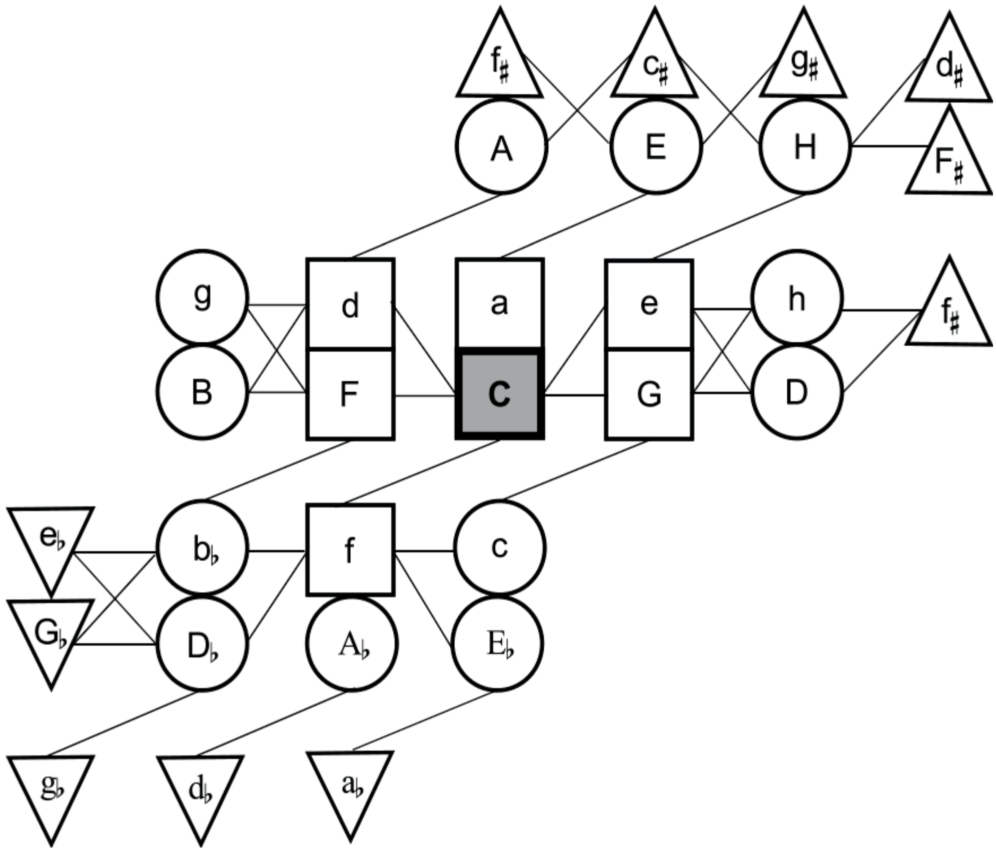
Having assumed that *n* is valid, we reproduce the whole modulation system that will be more generally symbolized by the difference in key signatures between the initial key and the rest of them.

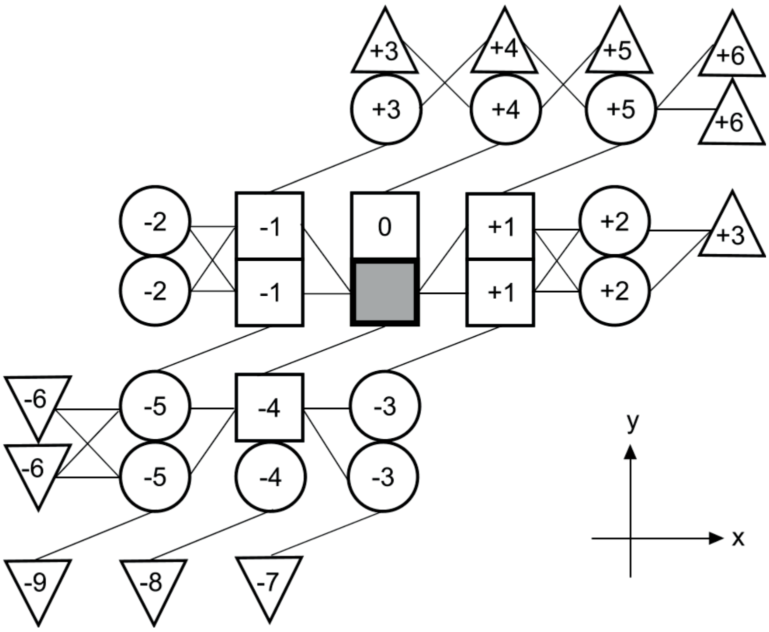
Scheme 9:

In case of initial minor, a chart is built up separately. As for the system by Rimsky-Korsakov, we will not go further than the third degree, the forth one being movement backward.

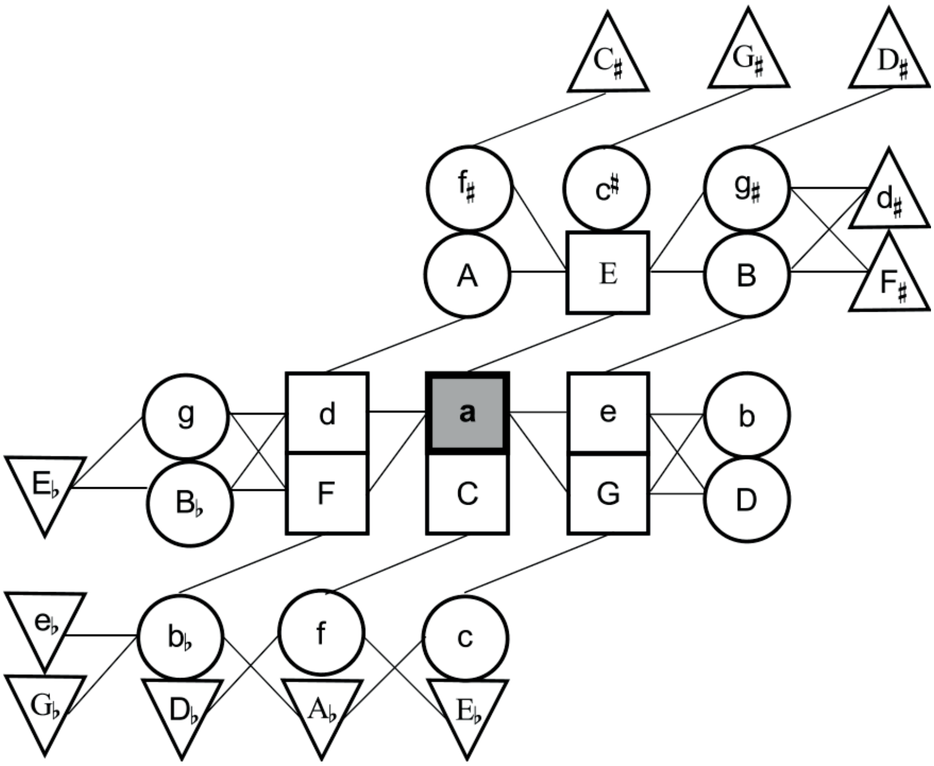
Enharmonically equal keys of the third degree can be symbolized by triangles with differently directed apices that would symbolize sharp and flat trends of modulation (schemes 11–14). Rimsky-Korsakov provides a different definition of the 2nd degree of relationship between the keys; however the end result (the list of appropriate keys) is the same as that given in my table. He defines the next degree not as the 3rd, but as a "modulation into distant keys". [5, pp. 81– 82, 86–87]

Scheme 11:

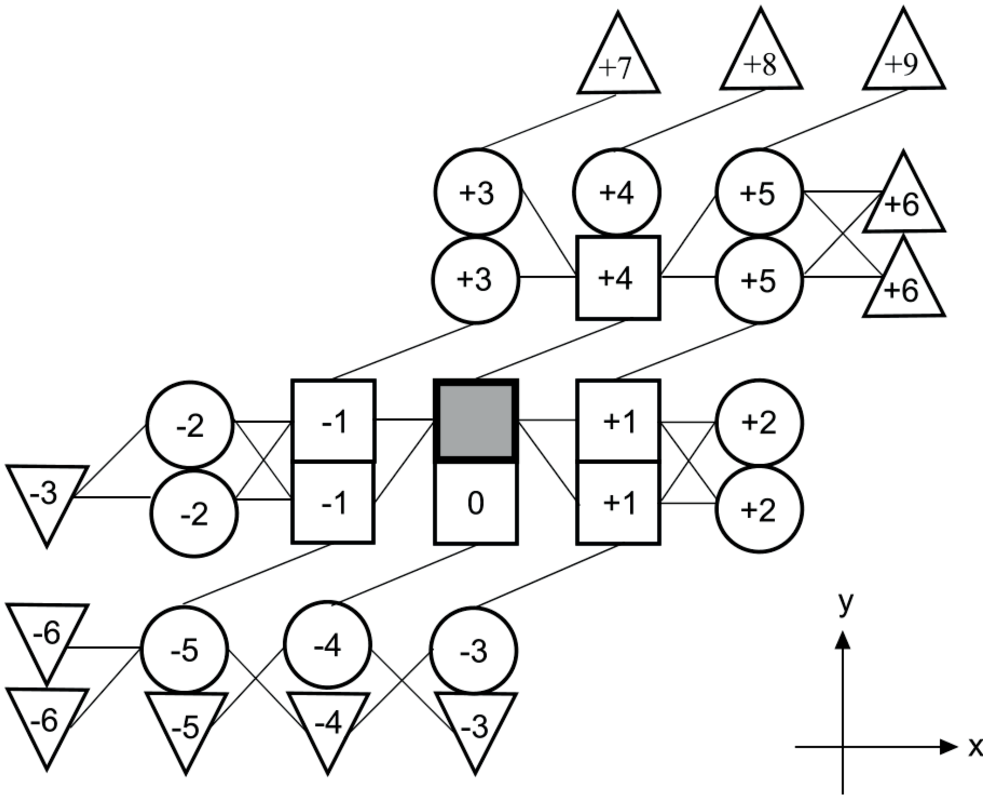



+3 +4 +5 +6
+3 +4 +5 +6
-2 -1 0 +1 +2 +3
-2 -1 +1 +2
-6 -5 -4 -3
-6 -5 -4 -3
-9 -8 -7
y
x

Scheme 13:


C♯ G♯ D♯
f♯ c♯ g♯ d♯
A E B F♯
g d a e b
E♭ B♭ F C G D
e♭ b♭ f c
G♭ D♭ A♭ E♭

Scheme 12:
Scheme 14:



Modulation paths are symbolized by the lines leading from the initial point to rectangles, from them – to circles, and from the circles – to triangles. Scheme 15 shows the system by Rimsky-Korsakov both for major and minor in two reference variants: there are common coordinates for minor (rightward and upward – plus, leftward and downward – minus) and mirror symmetrical coordinates for major (rightward and upward – minus, leftward and downward – plus).

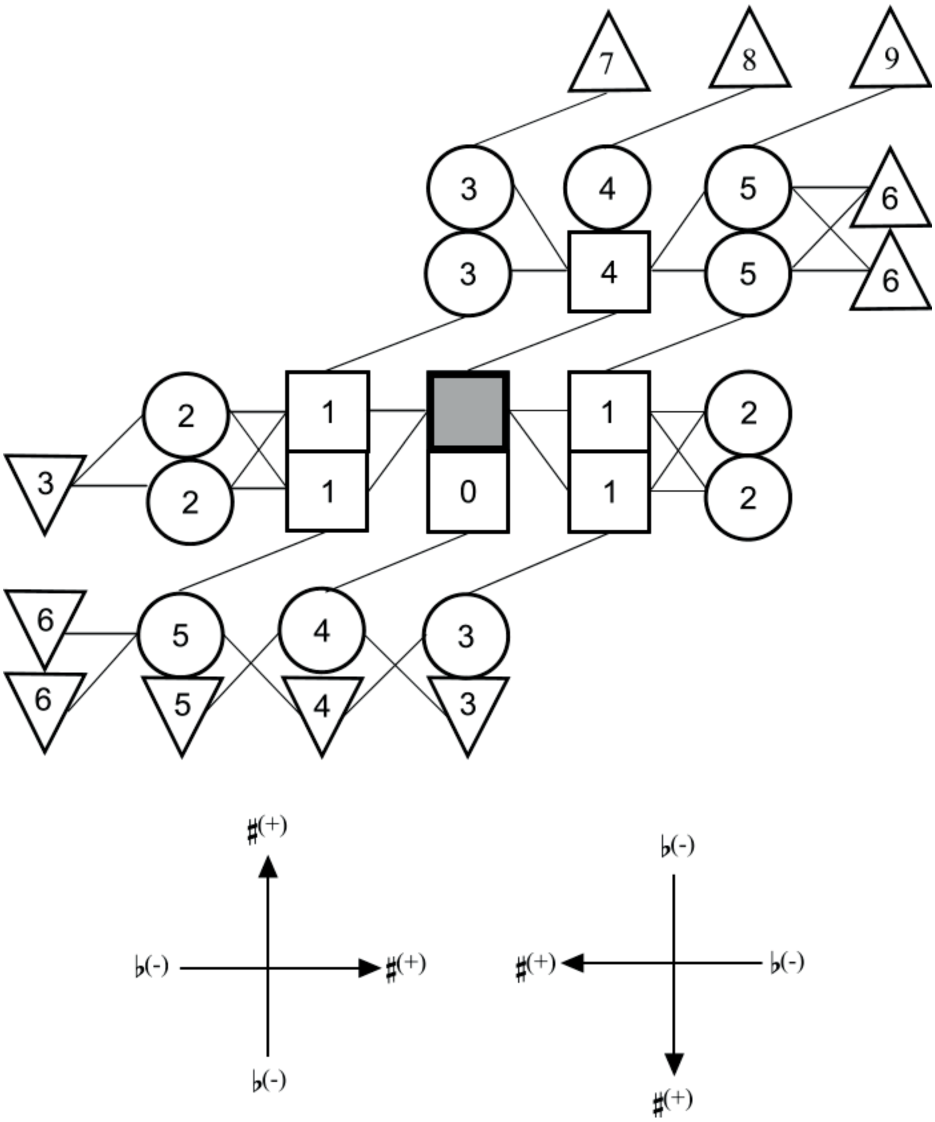
The variant of non-symmetrical major-minor system (see its description above) is given here in its finished appearance (schemes 16 and 17). It is suggested that readers take the initiative to do further creative work.

The pictures of modulation systems obtained are objects studied in mathematics by the graph theory ("shortest path problem"). For practical use, the whole graph can be turned into a stencil which is later applied to different parts of the field of keys, depending on from which key the modulation process is to start.

This is not a must, but is very demonstrative. Modulation graph numerical variant is enough to represent the whole system with any initial key.

In case of the symmetrical modulation system, it is not obligatory to have two different graphs to visually represent modulations from a major and from a minor. It will be sufficient to use one of these graphs (any), but with altered *x* and *y* axis direction from plus to minus and vice versa (see scheme 15).

Scheme 15:

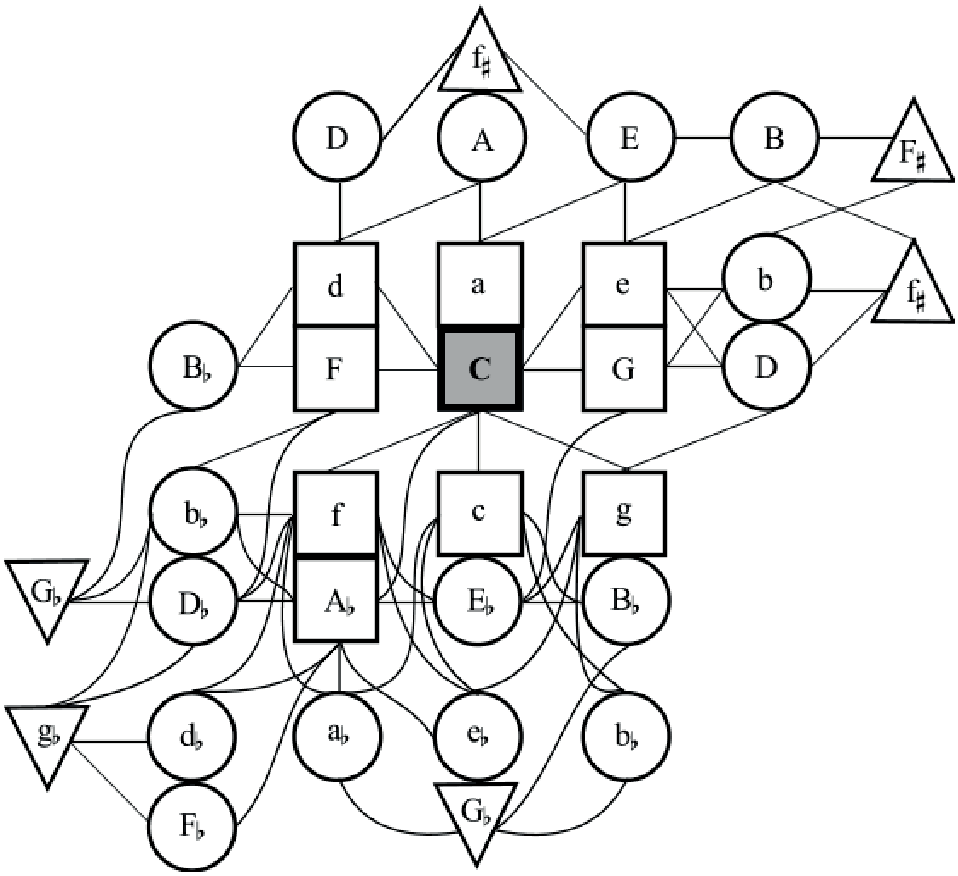


Minor Major

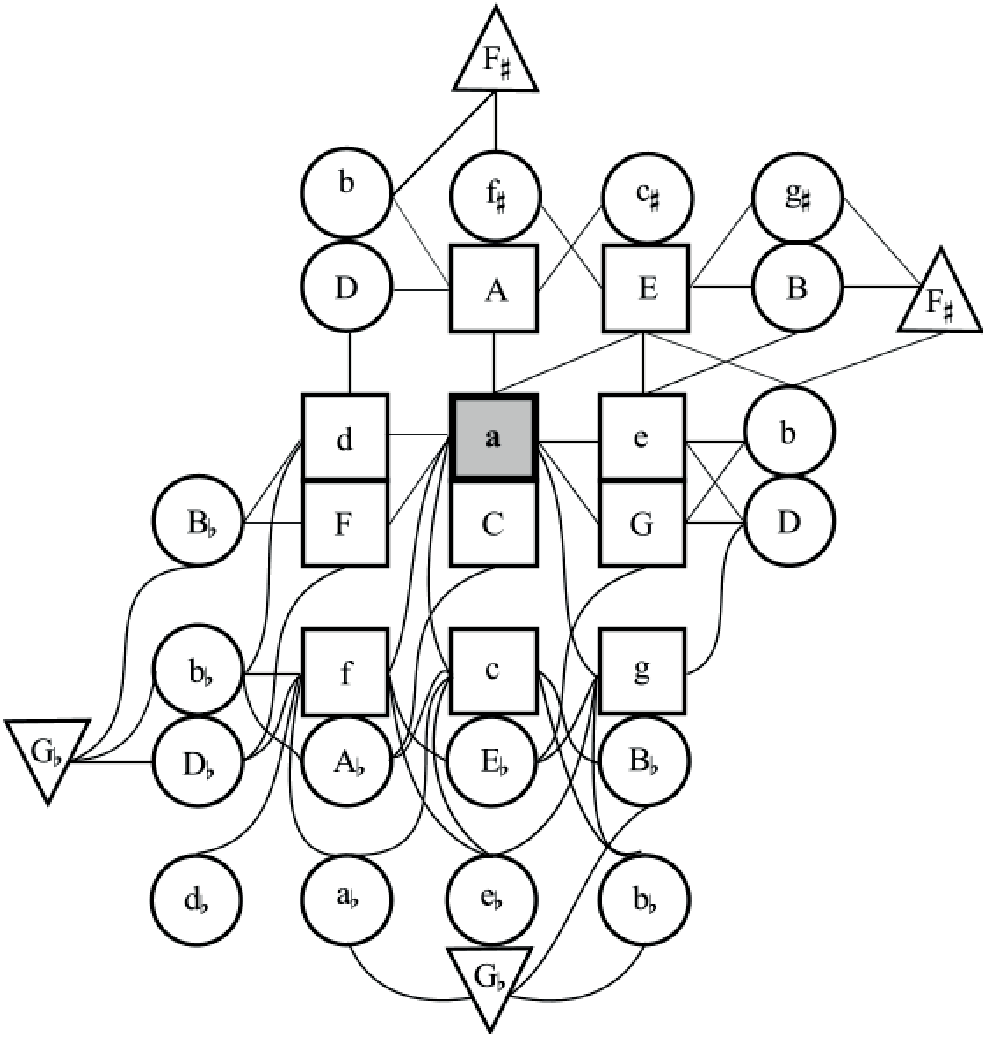
In the case of practical usage of a numerical graph, one should keep in mind the possibility of modulation into one and the same key in both directions – towards sharps and towards flats, provided that the index of modulation (difference in key signatures with indication of modulation direction in the form of sharp plus or flat minus) is not always obvious.

For example, the modulation index E major – F minor is not only –8, but +4 as well (E major – E sharp minor or F flat major – F minor). Therefore, it is necessary either to replace one of the keys to an enharmonically equal one or to remember that the sum of absolute values of both indices always equals 12, or else part of shortest modulation paths can be lost. This is true not for all modulations but only for the ones where the number of modulation steps either side is the same.

Scheme 16:



Scheme 17:



On the basis of the above, the minimum number of modulation steps in the system by Rimsky-Korsakov equals three both in F minor and E sharp minor, while in the proposed major-minor system in scheme 16 there were two steps from E major (which is to be placed into the initial point instead of C major) to F minor and three steps from E major to E sharp minor, that is the second path is not the shortest and should not be taken into consideration.

The practical task of finding all shortest paths from the key *X* to the key *Y* in this modulation system is solved in the following manner:

1) key signatures difference and modulation direction are determined (I call it modulation index here); there can be two indices of this kind, for example, +5 and –7; the sum of absolute values of both indices equals 12 (see above);

2) a box with the given indices is found in the numerical scheme, while keeping in mind that the upper square in a box symbolizes minor and the lower symbolizes major;

3) all paths leading from the initial point to the final one are examined, provided that the numbers in intermediate points indicate the difference in the key signatures between the final and intermediate keys.

Let me consider a modulation from B flat major to B minor in the system by Rimsky-Korsakov (see scheme 12):

1) a modulation index (difference in key signatures) from –2 (that is, two flats) to +2 (that is, two sharps) = +4 (but also –8);

2) the squares sought will be the triangle in the upper row (second from the left) and the triangle in the lower row (second from the left as well);

3) the shortest paths leading from the initial point to the final one will be the following:

- 0+4 (via a relative key and its major dominant, that is **B flat major**–*G minor–D minor*–**B minor**),
- –1+3 (via a relative key of a subdominant and its major dominant, that is **B flat major**–*C minor–G major*–**B minor**),
- +1+5 (via a relative key of a dominant and its major dominant, that is **B flat major**–*D minor–A major*–**B minor**),
- –4–4 (via a minor subdominant and its relative key, that is **B flat major**–*E flat minor–G flat major*–**B minor**).

The method proposed allows students to represent visually and cohesively different modulation systems that are subject to a modulation condition via a common accord and/or via an eponymous juxtaposition. In a real modulation process, movement does not have to follow the shortest paths. On the contrary, "supertonic" assumes a kind of a stroll around. But this is already a matter of an artistic task. Visual presentation of all real opportunities of the fastest withdrawal from the initial key may significantly facilitate the process of teaching this section of harmony, making knowledge more substantial.

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